Nr. 7

a) $f_t(x) = 10x \cdot e^{-\frac{1}{2}tx}$ $\quad t > 0$

Symmetrie

$f_t(-x) = 10 \cdot (-x)\, e^{-\frac{1}{2}t \cdot (-x)} = -10x \cdot e^{\frac{1}{2}tx} \neq f_t(x)$

$-f_t(-x) = -(-10x \cdot e^{\frac{1}{2}tx}) = 10x\, e^{\frac{1}{2}tx} \neq f_t(x)$

weder achsensymmetrisch zur y-Achse noch punktsymmetrisch zum Ursprung!

Achsenschnittpunkte

$f_t(0) = 10 \cdot 0 \cdot \underbrace{e^{-\frac{1}{2}t \cdot 0}}_{=1} = 0$

$f_t(x) = 0 \Leftrightarrow 10x \cdot e^{-\frac{1}{2}tx} = 0$

$\Leftrightarrow 10x = 0$ oder $\underbrace{e^{-\frac{1}{2}x} = 0}_{\text{nicht erfüllt!}}$

$\Leftrightarrow x = 0$

→ einziger Schnittpunkt mit den Achsen: $(0|0)$

Verhalten für $x \to \pm\infty$

$\lim\limits_{x \to +\infty} \underbrace{10x}_{+\infty} \cdot \underbrace{e^{-\frac{1}{2}tx}}_{\substack{\to 0 \\ (t>0)}} = 0$

also waagerechte Asymptote für $x \to +\infty$: $y = 0$

$\lim\limits_{x \to -\infty} \underbrace{10x}_{\to -\infty} \cdot \underbrace{e^{-\frac{1}{2}tx}}_{\substack{\to +\infty \\ (t>0)}} = -\infty$

Ableitungen $\quad f_t(x) = 10x \cdot e^{-\frac{1}{2}tx}$

$f_t'(x) = 10 \cdot e^{-\frac{1}{2}tx} + 10x \cdot e^{-\frac{1}{2}tx} \cdot (-\frac{1}{2}t)$

$= e^{-\frac{1}{2}tx} \cdot (10 - 5tx)$

$f_t''(x) = e^{-\frac{1}{2}tx} \cdot (-\frac{1}{2}t) \cdot (10 - 5tx) + e^{-\frac{1}{2}tx} \cdot (-5t)$

$= e^{-\frac{1}{2}tx} \cdot (-5t + 2{,}5t^2x - 5t)$

$= e^{-\frac{1}{2}tx} \cdot (2{,}5t^2x - 10t)$

$$f_t'''(x) = e^{-\frac{1}{2}tx}\cdot\left(-\tfrac{1}{2}t\right)\cdot(2{,}5t^2x-10t)+e^{-\frac{1}{2}tx}\cdot 2{,}5t^2$$
$$= e^{-\frac{1}{2}tx}\cdot(-1{,}25t^3x+5t^2+2{,}5t^2)$$
$$= e^{-\frac{1}{2}tx}\cdot(-1{,}25t^3x+7{,}5t^2)$$

<u>Extrema</u>

notw. Krit $\quad f_t'(x)=0$

$$\Leftrightarrow\quad e^{-\frac{1}{2}tx}\cdot(10-5tx)=0$$
$$\Leftrightarrow\quad \underbrace{e^{-\frac{1}{2}tx}}_{>0}=0 \quad \text{oder} \quad 10-5tx=0$$
$$\Leftrightarrow\quad 10=5tx \quad |:5t$$
$$\Leftrightarrow\quad \frac{10}{5t}=\frac{2}{t}=x$$

hinr. Krit. $\quad f_t''(x)\gtrless 0$

$$f_t''\left(\tfrac{2}{t}\right) = e^{-\frac{1}{2}t\cdot\left(\frac{2}{t}\right)}\cdot\left(2{,}5t^2\cdot\left(\tfrac{2}{t}\right)-10t\right)$$
$$= e^{-1}\cdot(5t-10t)$$
$$= \frac{1}{e}\cdot(-5t) = -\frac{5t}{e} < 0 \quad \left(\begin{matrix}\text{wegen}\\ t>0\end{matrix}\right)$$
$$\Rightarrow HP$$

y-Koord.

$$f_t\left(\tfrac{2}{t}\right) = 10\cdot\frac{2}{t}\cdot e^{-\frac{1}{2}t\cdot\left(\frac{2}{t}\right)} = \frac{20}{t}\cdot e^{-1} = \frac{20}{te}$$

also $HP\left(\frac{2}{t} \,/\, \frac{20}{t\cdot e}\right)$

<u>Wendepunkte</u>

notw. Krit: $\quad f_t''(x)=0$

$$\Leftrightarrow\quad e^{-\frac{1}{2}tx}\cdot(2{,}5t^2x-10t)=0$$
$$\Leftrightarrow\quad \underbrace{e^{-\frac{1}{2}tx}}_{>0}=0 \quad \underline{\text{oder}} \quad 2{,}5t^2x-10t=0$$
$$\Leftrightarrow\quad 2{,}5t^2x=10t \quad |:2{,}5t^2$$
$$\Leftrightarrow\quad x=\frac{10t}{2{,}5t^2}=\frac{4}{t}$$

hinr. Krit $\quad f_t'''(x)\neq 0$

$$f_t'''\left(\tfrac{4}{t}\right) = e^{-\frac{1}{2}t\cdot\left(\frac{4}{t}\right)}\cdot\left(-1{,}25t^3\cdot\left(\tfrac{4}{t}\right)+7{,}5t^2\right)$$
$$= e^{-2}\cdot(-5t^2+7{,}5t^2) = \frac{2{,}5t^2}{e^2} > 0 \quad [\neq 0]$$

also WP

y-Koord.

$$f_t\left(\frac{4}{t}\right) = 10\cdot\frac{4}{t}\cdot e^{-\frac{1}{2}t\cdot\frac{4}{t}} = \frac{40}{t}\cdot e^{-2} = \frac{40}{te^2}$$

also $WP\left(\frac{4}{t} \,\middle|\, \frac{40}{te^2}\right)$

b) Ortskurven

(1) der HPe: $\frac{2}{t} = x \Leftrightarrow 2 = x\cdot t \Leftrightarrow t = \frac{2}{x}$

$$g(x) = 10\cdot x\cdot e^{-\frac{1}{2}\cdot\frac{2}{x}\cdot x} = 10x\cdot e^{-1} = \frac{10}{e}\cdot x$$

(2) der WPe: $x = \frac{4}{t} \Leftrightarrow t\cdot x = 4 \Leftrightarrow t = \frac{4}{x}$

$$h(x) = 10\cdot x\cdot e^{-\frac{1}{2}\frac{4}{x}\cdot x} = 10x\cdot e^{-2} = \frac{10}{e^2}\cdot x$$

d) $F_1(x) = -20(x+2)\,e^{-\frac{1}{2}x}$

$$F_1'(x) = -20\cdot\left[1\cdot e^{-\frac{1}{2}x} + (x+2)\cdot e^{-\frac{1}{2}x}\cdot\left(-\frac{1}{2}\right)\right]$$

$$= -20\cdot\left[e^{-\frac{1}{2}x}\cdot\left(1+\left(-\frac{1}{2}x-1\right)\right)\right]$$

$$= -20\cdot e^{-\frac{1}{2}x}\cdot -\frac{1}{2}x = 10x\,e^{-\frac{1}{2}x} = f_1(x)$$

$$A = \int_0^{10} f_1(x)\,dx = \left[-20\cdot(x+2)\,e^{-\frac{1}{2}x}\right]_0^{10}$$

$$= -20(10+2)\cdot e^{-\frac{1}{2}\cdot 10} - \left(-20\cdot 2\cdot e^{0}\right)$$

$$= -20\cdot 12\cdot e^{-5} + 40$$

$$= -240\,e^{-5} + 40 = 40 - \frac{240}{e^5} = 38{,}38\ldots$$

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